1986-09

A comparison of four estimators of a first order autoregressive process

Horn, Joseph A. Jr.

http://hdl.handle.net/10945/21718

NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

A COMPARISON OF FOUR ESTIMATORS

OF

A FIRST ORDER AUTOREGRESSIVE PROCESS

by

Joseph A. Horn Jr.

September 1986

Thesis Advisor: D. C. Boger

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ECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

a REPORT SECURITY CLASSIFICATION: UNCLASSIFIED

1b. RESTRICTIVE MARKINGS:

2a SECURITY CLASSIFICATION AUTHORITY:

2b DECLASSIFICATION / DOWNGRADING SCHEDULE:

3 DISTRIBUTION / AVAILABILITY OF REPORT: Approved for public release; distribution is unlimited.

PERFORMING ORGANIZATION REPORT NUMBER(S):

5 MONITORING ORGANIZATION REPORT NUMBER(S):

a NAME OF PERFORMING ORGANIZATION: Naval Postgraduate School

6b OFFICE SYMBOL *(If applicable)*: Code 55

7a NAME OF MONITORING ORGANIZATION: Naval Postgraduate School

c ADDRESS *(City, State, and ZIP Code)*: Monterey, California 93943-5000

7b ADDRESS *(City, State, and ZIP Code)*: Monterey, California 93943-5000

a NAME OF FUNDING / SPONSORING ORGANIZATION:

8b OFFICE SYMBOL *(If applicable)*:

9 PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER:

c ADDRESS *(City, State, and ZIP Code)*:

10 SOURCE OF FUNDING NUMBERS

PROGRAM ELEMENT NO	PROJECT NO	TASK NO	WORK UNIT ACCESSION NO

1 TITLE *(Include Security Classification)*: A COMPARISON OF FOUR ESTIMATORS OF A FIRST ORDER AUTOREGRESSIVE PROCESS

2 PERSONAL AUTHOR(S): Horn, Joseph A. Jr.

3a TYPE OF REPORT: Master's Thesis

13b TIME COVERED: FROM ______ TO ______

14 DATE OF REPORT *(Year, Month, Day)*: 1986 September

15 PAGE COUNT: 49

6 SUPPLEMENTARY NOTATION:

7 COSATI CODES

FIELD	GROUP	SUB-GROUP

18 SUBJECT TERMS *(Continue on reverse if necessary and identify by block number)*: Autoregression, Autocorrelation, Durbin-Watson, Prais-Winsten, Theil-Nagar, Beach-MacKinnon

9 ABSTRACT *(Continue on reverse if necessary and identify by block number)*

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0 DISTRIBUTION / AVAILABILITY OF ABSTRACT: ☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT ☐ DTIC USERS

21 ABSTRACT SECURITY CLASSIFICATION: UNCLASSIFIED

2a NAME OF RESPONSIBLE INDIVIDUAL: Dan C. Boger

22b TELEPHONE *(Include Area Code)*: (408) 646-3228

22c OFFICE SYMBOL: Code 53Bo

D FORM 1473, 84 MAR

83 APR edition may be used until exhausted
All other editions are obsolete

SECURITY CLASSIFICATION OF THIS PAGE

A Comparison of Four Estimators of a First Order Autoregressive Process

by

Joseph A. Horn Jr.
Lieutenant, United States Navy
B.S.M.E., United States Naval Academy, 1980

Submitted in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

NAVAL POSTGRADUATE SCHOOL
September 1986

ABSTRACT

Econometricians must choose between many methods for estimating ρ, the autocorrelation coefficient, in a first order autoregressive process. This thesis examines the performance of four estimators in a Monte Carlo simulation. The methods examined are Durbin-Watson, Beach-MacKinnon, Theil-Nagar and Prais-Winsten. The autocorrelation coeficient, ρ, was varied from .2 to .9 and each method provided estimates of ρ and β, the regression coefficient, for 1000 replications. The results presented here are similar to those found in previous comparisons. Specifically, Ordinary Least Squares was found to be an efficient estimator of β when autocorrelation is present only to a slight degree. Of the four estimators examined, the performance of Theil-Nagar proved superior in estimating both ρ and β for small values of the correlation coeficient. Beach-MacKinnon, on the other hand, while containing a large bias in the estimation of ρ, is the more efficient estimator of β for large values of ρ.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

I. INTRODUCTION

A. BACKGROUND

Autocorrelation exists in a regression model when the error terms are no longer independent but are correlated. In the examination of time series data autocorrelation is a common phenomenon and can lead to problems if Ordinary Least Squares (OLS) estimation procedures are used. The purpose of this thesis is to examine and compare four different estimates of the autocorrelation coefficient, ρ, the estimation of which is essential to the resolution of OLS deficiencies. The four estimators to be examined are the Durbin-Watson, Theil-Nagar, Beach-MacKinnon, and Prais-Winsten.

B. PROBLEM STATEMENT

In the standard regression model $y = X\beta + e$, y is a Tx1 vector of observations of a dependent variable, X is a TxK design matrix and β is a Kx1 vector. The variable e is a Tx1 vector of unobservable random errors with $E(e) = 0$ and covariance matrix, $E(ee') = \sigma^2 I_T$. Thus, in the standard model, the random vector e contains elements which are pairwise uncorrelated with identical means and variances. In the presence of autocorrelation this strong assumption is violated. That is, the error terms are no longer independent but are correlated. The regression model becomes,

$$y_t = X_t\beta + e_t \qquad t = 1,2,\ldots,T \qquad \text{(eqn 1.1)}$$

$$\text{where} \quad e_t = \rho\, e_{t-1} + v_t,$$
$$E(v_t) = 0, \text{ and}$$
$$E(vv') = \sigma^2 I\ .$$

This is known as a first order autoregressive or AR(1) process. As illustrated by equation 1.1, e_t is expressed linearly in terms of the e_{t-1} and another random error term v_t. The assumption of zero mean and constant variance provides v_t with all the nice properties of e_t in the standard model. This process may occur for a variety of reasons, some of which are:

1. *Omitted explanatory variables.* If a correlated explanatory variable has been excluded from the design matrix its exclusion will be reflected in the correlation of the random variable e.
2. *Mispecification of the mathematical form of the model.* If the wrong mathematical relationship is chosen the values of e may be dependent.

3 *Interpolations in the statistical observations.* If the observational data is smoothed autocorrelation may result.

4 *Mispecification of the true random error.* Dependence among the error terms may occur naturally. [Ref. 1:p. 204]

Utilizing OLS to estimate the regression coefficient, β, in the presence of an AR(1) process can lead to problems. Generally, there are two consequences to consider. The first is that the OLS estimator of the coefficients will be unbiased but will not be very efficient. The second consequence is that the OLS variance estimator is biased. For these reasons it is useful to investigate other methods to estimate β [Ref. 2:p. 439].

C. ESTIMATORS

When ρ is known, the process is easily accounted for using Generalized Least Squares or Weighted Least Squares methods [Ref. 3]. However, the usual situation is that ρ is unknown and must be estimated. A number of methods have been proposed to estimate ρ and properly account for OLS deficiencies in estimating β. Chapter 2 will develop the four estimators mentioned above and examine the autocorrelation process.

D. SIMULATION

Each of the estimators considered here have the same asymptotic properties therefore any decision on which one to use must be based on small sample analysis and Monte-Carlo evidence. Therefore, a simulation will be created in which the data is generated according to guidelines presented in previous studies with equation 1.1 as the model. The actual values of ρ will be varied from .2 to .9. The four estimation techniques will then provide estimates of ρ and β for 1000 replications.

E. MEASURE OF EFFECTIVENESS

To provide an indication of which estimator performs best the mean square error of both $\hat{\rho}$ and $\hat{\beta}$ will be estimated for each estimator. Prior results for different sets of estimators indicate that no one estimator will prove superior over the entire range of ρ but that one or two may out perform the others over specific intervals.

II. ESTIMATION

A. GENERAL

This chapter attempts to explore the theory behind both the first order process and four estimators developed to properly account for it. Three of these (Durbin-Watson, Thiel-Nagar, and Prais-Winsten) are categorized as estimated generalized least squares estimators. The fourth (Beach-MacKinnon) is a maximum likelihood estimator.

B. PROCESS

The first order process can be written as

$$y_t = X_t\beta + e_t \qquad t = 1,2,\ldots,T \qquad \text{(eqn 2.1)}$$

where $e_t = \rho\, e_{t-1} + v_t$,

$E(v_t) = 0$,

$E(vv') = \sigma^2 I$,

$E(v_t^2) = \sigma_v^2$, and

$E(v_t v_s) = 0$ for $s \neq t$.

The parameter ρ is generally unknown and along with β must be estimated. The statistical properties of the random error, v, listed in equation 2.1 are identical to those listed for e in the general linear model. The statistical properties of e under these new assumptions are quite different. Judge [Ref. 4:p. 438] shows that

$$E(e_t) = \sum_{i=1}^{\infty} \rho^i E(v_{t-i}) = 0 \qquad \text{(eqn 2.2)}$$

and

$$E(e_t^2) = \sigma_e^2 = \sigma_v^2/(1-\rho)^2 \ . \qquad \text{(eqn 2.3)}$$

The covariance between errors s periods apart is no longer zero and is given by

$$E(e_t e_{t-s}) = E(e_{t+s} e_t) = (\rho^s \sigma^2{}_v)/(1-\rho^2) \ . \qquad \text{(eqn 2.4)}$$

The covariance matrix for e is now easily written as

$$\Phi = E(ee') = \qquad \text{(eqn 2.5)}$$

$$= \sigma^2_v/(1-\rho^2) \begin{vmatrix} 1 & \rho & . & . & . & \rho^{T-1} \\ \rho & 1 & . & . & . & \rho^{T-2} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ \rho^{T-1} & \rho^{T-2} & . & . & . & 1 \end{vmatrix}$$

or utilizing the following convention,

$$\Phi = \sigma_v{}^2\Psi \qquad \text{(eqn 2.6)}$$

where Ψ =

$$= 1/(1-\rho^2) \begin{vmatrix} 1 & \rho & \rho^2 & . & . & . & \rho^{T-1} \\ \rho & 1 & \rho^2 & . & . & . & \rho^{T-2} \\ \rho^2 & \rho & 1 & . & . & . & \rho^{T-3} \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ \rho^{T-1} & \rho^{T-2} & \rho^{T-3} & . & . & . & 1 \end{vmatrix} .$$

Thus, the assumptions made about the error term, e, in the standard linear model no longer hold for the autoregressive case. Specifically, due to autocorrelation the error covariance matrix is no longer written as $\sigma^2 I$ but is now $\sigma^2_v\Psi$.

When an attempt is made to perform a least squares fit to the data in the presence of an AR(1) process there are two problems to consider.

1 The least squares estimator $\hat{\beta} = (X'X)^{-1}X'y$ will be unbiased but will not be very efficient.

2 The least squares covariance matrix $\hat{\sigma}^2(X'X)^{-1}$ with $\hat{\sigma}^2 = (y-Xb)'(y-Xb)/(T-K)$ will be a biased estimator of the variance of $\hat{\beta}$.

In the presence of positive autocorrelation Judge [Ref. 4:p. 439] notes that with OLS estimation the bias of the standard error of $\hat{\beta}$ will very likely appear as an

underestimate. Park and Mitchell [Ref. 5:p. 16] warn that OLS seriously underestimates the variance of $\hat{\beta}$ for $\rho > 0.4$. This understatement makes the estimates themselves appear much more significant than they actually are and makes hypothesis testing of the slope coefficients unreliable.

C. METHODS OF ESTIMATION

1. Generalized Least Squares Estimation

When apriori information is available about Ψ, the most convenient estimate for the regression coefficient, $\hat{\beta}$, is obtained by applying least squares estimation techniques to the transformed model,

$$Y^* = X^*\beta + e^* \qquad \text{(eqn 2.7)}$$

where $Y^* = PY$

$X^* = PX$

$e^* = Pe.$

The transformation matrix P is the TxT matrix

$$P = \begin{vmatrix} \sqrt{1-\rho^2} & 0 & 0 & 0 & . & . & . & 0 \\ -\rho & 1 & 0 & 0 & . & . & . & 0 \\ 0 & -\rho & 1 & 0 & . & . & . & 0 \\ 0 & 0 & -\rho & 1 & . & . & . & 0 \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ 0 & 0 & 0 & . & . & . & -\rho & 1 \end{vmatrix}$$

where $P'P = \Psi^{-1}$.

This method is known as the Generalized Least Squares (GLS) estimation.

2. Estimated Generalized Least Squares

The usual case is that ρ is unknown and must be estimated. Once an estimate for ρ ($\hat{\rho}$) is computed one can substitute $\hat{\rho}$ into the P matrix and proceed with the GLS method outlined above. This is known as Estimated Generalized Least Squares (EGLS) estimation. The computational form of the alternative estimators for ρ discussed are as follows:

a. Durbin-Watson

The statistic

$$d = \sum_{t=2}^{T} (\hat{e}_t-\hat{e}_{t-1})^2 / \sum_{t=1}^{T} \hat{e}_t^2 \qquad t=1,\ldots,T \qquad \text{(eqn 2.8)}$$

where $\hat{e}_t = y_t - X_t\hat{\beta}$

is often used to test for first order autoregressive errors. As the number of observations (T) increases it can be demonstrated that d approaches the least squares estimator of ρ or

$$\hat{\rho} = 1 - (d/2) . \qquad \text{(eqn 2.9)}$$

The Durbin-Watson statistic is provided by most least squares computer packages and is very easy to use. It also is an example of a two-stage estimator. That is, it first estimates the correlation parameter and then uses this estimate to compute the generalized least squares estimates for β.

b. Theil-Nagar

A modification of the Durbin-Watson estimator suggested by Henri Theil and A. L. Nagar is

$$\hat{\rho} = (T^2(1-(d/2)) + K^2) / (T^2 - K^2). \qquad \text{(eqn 2.10)}$$

Theil and Nagar claim that this estimator is an improvement over Durbin-Watson if the first and second differences of the explanatory variables are small when compared to their corresponding ranges [Ref. 6]. Like Durbin-Watson, it also is a two-stage estimator.

c. Prais-Winsten

A minimum sum of squares approach to estimating ρ yields,

$$\hat{\rho} = \sum_{t=2}^{T} \hat{e}_t\hat{e}_{t-1} / \sum_{t=1}^{T-1} \hat{e}_t 2 \qquad t=1,\ldots,T \qquad \text{(eqn 2.11)}$$

where $\hat{e}_t = y_t - X_t\hat{\beta}$.

This estimator can be employed in both a two step and an iterative procedure. This paper, however, considers only the following iterative form:

1. Set $\hat{\rho} = 0$.
2. Transform the variables in accordance with the transformation matrix and equation 2.7.
3. Calculate the least squares estimate of β conditional on $\hat{\rho}$.
4. Calculate the estimate of ρ conditional on $\hat{\beta}$ by using equation 2.11.
5. If the absolute difference in $\hat{\rho}$ from the previous iteration is sufficiently small (less than 0.00001) stop. If not go to step 2. [Ref. 7:p. 2]

3. Maximum Likelihood Estimation

A maximum likelihood (ML) estimator is the value of θ which maximizes the value of the likelihood function $L(\theta)$. Under the assumption that Y has a multivariate normal distribution with mean $X\beta$ and covariance matrix $\sigma^2\Psi$, the likelihood function is

$$L(\beta,\rho,\sigma^2) = C - (1/2\sigma^2_v)(y-X\beta)'\Psi^{-1}(y-X\beta) \qquad \text{(eqn 2.12)}$$
$$\text{where } C = -(T/2)\ln\sigma^2_v + (1/2)\ln(1-\rho^2)\,.$$

The ML estimators for β, ρ, and σ_v^2 are those values for which,

$$\partial L/\partial \beta = 0\ ,\quad \partial L/\partial \rho = 0\ ,\quad \partial L/\partial \sigma^2_v = 0\ . \qquad \text{(eqn 2.13)}$$

Solutions to equations 2.13 are very difficult to derive. Beach and MacKinnon [Ref. 8:p. 54] use an ML estimator for σ_v^2 and substitute into equation 2.12. The result is the concentrated likelihood function,

$$L(\beta,\rho) = K-(T/2)\ln((y-X\beta)'\Psi^{-1}(y-X\beta)(1-\rho^2)^{1/T}) \qquad \text{(eqn 2.14)}$$
$$\text{where } K = (T/2)\ln(T)-(T/2)\,.$$

They suggest maximizing $L(\beta,\rho)$ with respect to β with ρ held constant and then to maximize with respect to ρ with β held constant. An algorithm to derive this ML estimate is

1. Set $\hat{\rho} = 0$.

2. Transform the variables in accordance with equation 2.7.

3. Calculate the least squares estimate of β conditional on $\hat{\rho}$.

4. Calculate the ML estimate of ρ conditional on $\hat{\beta}$ by solving a cubic equation of the untransformed residuals. (see [Ref. 8] for details)

5. If the absolute difference in $\hat{\rho}$ from the previous iteration is sufficiently small (less than 0.00001) stop. If not, go to step 2 [Ref. 7]. (Note: The same procedure was employed for iterative Prais-Winsten method except that equation 2.11 was used to estimate ρ.)

This is not a comprehensive listing of all available estimators for a first order process. Other estimators are listed in Judge [Ref. 4].

III. COMPARISON

A. GENERAL

The finite sampling properties of the estimators listed here have not been derived. Choice of which estimator to use might be based on evidence obtained from Monte Carlo simulations. This chapter explains a simulation used and provides a synopsis of comparisons reported in the literature.

B. PREVIOUS COMPARISONS

There have been a number of studies of estimators for ρ . Each has concluded that OLS has serious deficiencies in the presence of autocorrelation. The majority of these papers have settled on two points. First, particularly in small sample sizes ($T < 50$) it is best to use estimators that consider all T observations. Rao and Grilitches concluded that using estimators such as Cochrane-Orcutt that ignore the first observation can lead to a substantial loss of efficiency [Ref. 9:p. 269]. These results were further substantiated by Beach and MacKinnon. In an attempt to develop a computationally efficient algorithm to maximize the likelihood function they discovered (for $\rho =$ 0.6, 0.8, 0.99) significant gains in efficiency to be made by employing the first observation. Some of these gains are in the neighborhood of 700 percent [Ref. 8:p. 55]. Park and Mitchell concluded that retention of this first observation substantially reduces the risk of collinearity as ρ approaches 0.9 [Ref. 5:p. 10]. Kobayashi verified theoretically the experimental results of Park and Mitchell. By computing the asymptotic variances of several estimators he demonstrated that the loss of efficiency of the Cochrane-Orcutt method was due primarily to ignoring the first observation. [Ref. 10:p. 951].

The second point is that the Prais-Winsten solution techniques outperform many comparable estimators of the correlation parameter. Spitzer concluded that Prais-Winsten "appeared to be the best of all the two stage estimators." [Ref. 11:p. 44]. Park and Mitchell in a later study comparing Beach-MacKinnon with the iterative Prais-Winsten estimator concluded that the iterative Prais-Winsten performs "appreciably better in estimating the autocorrelation coefficient ρ" [Ref. 7:p. 5].

Although there were no studies found specifically comparing the four estimators presented here, each has demonstrated a superiority to OLS in the presence of a first order process.

C. MODEL AND DATA GENERATION

Equation 2.1 was utilized as the model with the first term in the vector e generated in the following fashion,

$$e_1 = v_1/(1-\rho)^{1/2}. \qquad \text{(eqn 3.1)}$$

In order to conform with previous comparisons, the data utilized in this experiment is identical to that used in Beach and MacKinnon [Ref. 8]. Two sample sizes of 20 and 50 observations were used. The untrended explanatory variable, X, was drawn from N(0, 0.0625) and the random error, v_t, was drawn from N(0, 0.0036). Although autocorrelation in theory may be positive or negative, in econometric data it is almost always positive [Ref. 1:p. 201]. For this reason ρ was varied from 0.2 to 0.9.

D. VALIDATION

The data generation program was checked to ensure the normality of e using the Chi Square Goodness of Fit test. The normality assumption was accepted at a 0.3684 level. Finally, in order to ensure each estimator performed properly the random portion of the model, specifically the random variable V, was removed. This allowed the estimators to function in a deterministic fashion. Data were then generated and submitted to each estimator for values of ρ equal 0.2, 0.6, 0.8. The results are presented in Table I , illustrate that the estimators are functioning properly.

E. SIMULATION

For each run the values of the regression coefficients, β_0 and β_1, were set to 1 and 1. The variables X_t and v_t were drawn from the normal distributions discussed earlier. The dependent variable y_t was calculated using equation 2.1. Since the ultimate objective was to generate residuals to send to the four estimation routines, a regression was then performed of y on X and residuals calculated using,

$$\hat{e}_t = y_t - x_t\hat{\beta} \qquad t = 1,2,\ldots,T. \qquad \text{(eqn 3.2)}$$

TABLE I

ESTIMATES OF AUTOCORRELATION COEFFICIENT

ρ	DW	TN	PW	BM
.2	.19	.19	.19	.19
.6	.59	.60	.58	.60
.8	.78	.80	.77	.80

The values of the residuals were then sent to each estimation routine. Estimates of β and ρ were determined for values of ρ equal to .2, .3, .4, .5, .6, .7, .8, and .9. Each estimate was replicated 1000 times for the sample sizes of 20 and 50.

F. MEASURES OF EFFECTIVENESS

In order to compare the performances of the estimators, two MOE's were used. The mean square error (MSE) of $\hat{\rho}$ was estimated for each estimator. This represents the expected squared error made in estimating ρ. The following computational form of MSE was used,

$$\sum_{i=1}^{1000} (\rho - \rho_i)^2/1000 \qquad i = 1,\ldots,1000. \qquad \text{(eqn 3.3)}$$

The successive values of MSE of $\hat{\rho}$ were then plotted against the actual ρ to examine performance over the range of ρ.

The second MOE examined the relative efficiencies of the regression coefficient as defined in [Ref. 7:p. 7]. A ratio of MSE of $\hat{\beta}$ for a particular estimate to the MSE of $\hat{\beta}$

for the OLS estimate allows the examination of the relative gains in using particular techniques over OLS. Since the proper estimation of β is paramount the efficiency of $\hat{\beta}$ is predetermined to be the most important MOE.

IV. RESULTS AND CONCLUSIONS

A. GENERAL

The major emphasis of this thesis was to examine the performance of four estimators of the autocorrelation coefficient, ρ, for a first order autoregressive process. The estimators examined were Durbin-Watson, Theil-Nagar, Prais-Winsten, and Beach-MacKinnon.

A Monte-Carlo simulation was performed for the following values of ρ: .2, .3, .4, .5, .6, .7, .8, and .9. Each run was replicated 1000 times for sample sizes of 20 and 50. The results are recorded in Table II . Irrespective of sample size, each of the methods underestimate the true value of ρ but as the number of observations is increased from 20 to 50 the bias reduces. As was expected, no one estimator uniformly outperforms the others. In both sample sizes, the two stage estimators (Durbin-Watson and Theil-Nagar) achieve better results for small ρ. As the value of ρ increases, the iterative methods (Prais-Winsten and Beach-MacKinnon) perform best. With $T=20$ this transition occurs at $\rho=.6$ while at 50 observations it occurs earlier at $\rho=.4$.

The discussion of the results will be divided into two sections. The measures of effectiveness, as defined in Chapter 3 will first be applied to the simulation results for $T=20$. This will be followed by an identical approach when the sample size is increased to 50.

B. SAMPLE SIZE 20

Since performance of an estimator is roughly indicated by its mean and variance, mean square error (MSE) of each $\hat{\rho}$ over the entire sample size was estimated. The results of these calculations are presented in Table III along with a plot of MSE of $\hat{\rho}$ versus actual values of ρ in Figure 4.1 . They again indicate that the Theil-Nagar and Durbin-Watson estimators are better for smaller values of ρ ($\rho<.6$) and as ρ increases the Prais-Winsten ρ emerges as the best. On the basis of Figure 4.1 alone, Beach-MacKinnon's performance is clearly inferior. However, in examining the efficiency of each estimator in Table IV, Beach-MacKinnon proves to be the most efficient in estimating β over the widest range of ρ. The tie in Figure 4.1 between Theil-Nagar and Durbin-Watson is resolved in Table IV with Theil-Nagar proving to

TABLE II

ESTIMATES OF AUTOCORRELATION COEFFICIENT

Sample Size 20

ρ	DW	TN	PW	BM
.2	.158	.162	.113	.107
.3	.234	.239	.205	.193
.4	.310	.316	.296	.279
.5	.385	.392	.387	.365
.6	.460	.467	.478	.450
.7	.533	.542	.567	.535
.8	.603	.613	.655	.617
.9	.667	.681	.741	.697

Sample Size 50

ρ	DW	TN	PW	BM
.2	.173	.161	.170	.178
.3	.279	.253	.270	.270
.4	.360	.340	.360	.359
.5	.451	.432	.463	.453
.6	.544	.523	.559	.547
.7	.630	.610	.651	.640
.8	.726	.700	.747	.731
.9	.812	.796	.839	.820

be uniformly more efficient than Durbin-Watson. Table IV also demonstrates that for $\rho = .2$ OLS is at least as efficient as three of the four estimators.

C. SAMPLE SIZE 50

The results of the MSE calculations for T = 50 are recorded in Table V along with a plot of MSE of ρ versus the actual values of ρ in Figure 4.2. The Durbin-Watson and Theil-Nagar estimators again perform the best for smaller values of ρ ($\rho < .4$) and as ρ increases the Beach-McKinnon and Prais-Winsten estimators of ρ contain the smallest MSE.

Once again even though the Prais-Winsten ρ has a smaller MSE than Beach-MacKinnon, Table VI illustrates that Beach-McKinnon is a uniformly more efficient estimator of the slope coefficient. For the smaller values of ρ ($\rho < .4$) Theil-Nagar is more efficient than Durbin-Watson. Table VI also illustrates that OLS is at least as efficient as any of the other estimators when ρ is small.

D. SUMMARY

As was found in previous studies when autocorrelation is present only to a slight degree ($\rho < .2$) the OLS estimator provides an efficient estimate for the regression coefficient, β. As the process becomes more significant however, all the estimators outperform the OLS solution. In both sample sizes the performance of Theil-Nagar and Durbin-Watson are nearly identical with respect to the MSE of ρ. However, when efficiency of the slope coefficient estimate is examined, Theil-Nagar proves to be the better 2 stage estimator. Park and Mitchell [Ref. 7:p. 4] found that Prais-Winsten performs better in estimating β. The results presented here tend to dispute that finding. For while Prais-Winsten has a uniformly smaller MSE of ρ, Beach-MacKinnon provides the most efficient estimator of β . Spitzer, on the other hand [Ref. 11:p. 44], which ranked two stage estimators as being the best for values of ρ between .2 and .5, mirrors the results produced here. Apriori knowledge of the neighborhood of ρ will be helpful in selecting the appropriate estimation method. For both sample sizes Theil-Nagar appears to be the best for small values of ρ. Beach-MacKinnon, while containing a larger bias for ρ than does Prais-Winsten, is a much more efficient estimator of the slope coefficient for larger values of ρ.

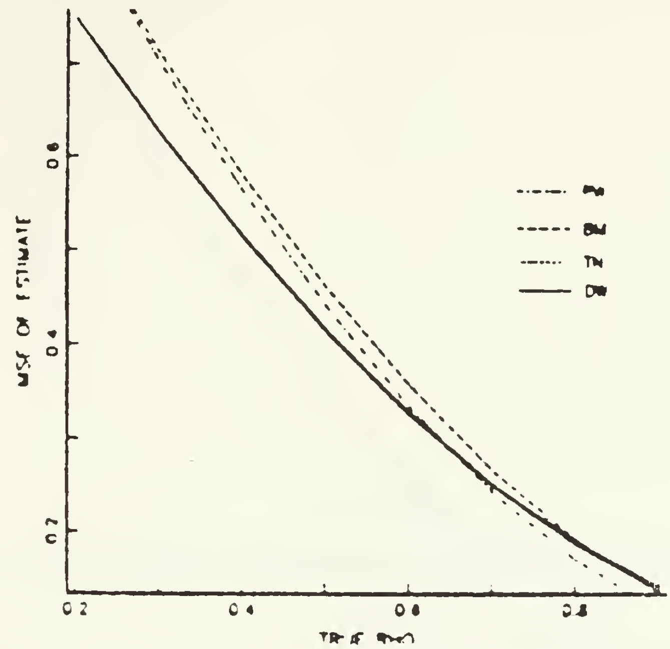

Figure 4.1 Estimated mean square error of ρ vs. ρ (sample size = 20).

TABLE III
DATA PRESENTED IN FIGURE 4.1

Sample Size 20

ρ	MSEDW	MSETN	MSEPW	MSEBM
.2	.7494	.7485	.8557	.8594
.3	.6268	.6244	.7004	.7115
.4	.5156	.5124	.5621	.5787
.5	.4159	.4126	.4400	.4604
.6	.3278	.3250	.3342	.3566
.7	.2519	.2495	.2433	.2662
.8	.1891	.1860	.1698	.1916
.9	.1407	.1357	.1141	.1331

TABLE IV

EFFICIENCY OF REGRESSION COEFFICIENT ESTIMATES

Sample Size 20

ρ	MSEβ (DW) / MSEβ (OLS)	MSEβ (TN) / MSEβ (OLS)	MSEβ (PW) / MSEβ (OLS)	MSEβ (BM) / MSEβ (OLS)
.2	1.004	.9794	1.035	1.041
.3	.9228	.8967	.9442	.9515
.4	.8218	.7929	.8325	.8342
.5	.7082	.6751	.7024	.6959
.6	.5864	.5484	.5652	.5515
.7	.4610	.4207	.4329	.4135
.8	.3359	.3020	.3093	.2870
.9	.2253	.2087	.2077	.1892

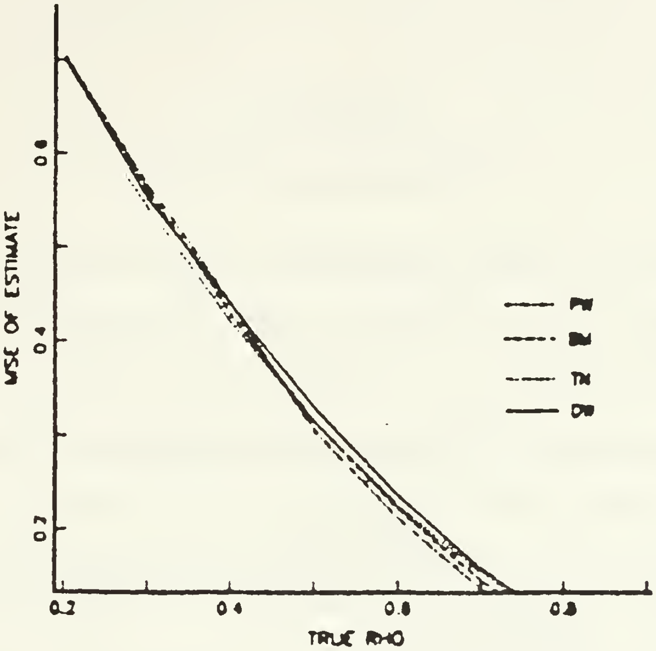

Figure 4.2 Estimated mean square error of $\hat{\rho}$ vs. ρ (sample size = 50).

TABLE V

DATA PRESENTED IN FIGURE 4.2

Sample Size 50

ρ	MSEDW	MSETN	MSEPW	MSEBM
.2	.7010	.6766	.7055	.7065
.3	.5500	.5399	.5653	.5578
.4	.4383	.4196	.4396	.5578
.5	.3298	.3151	.3056	.3156
.6	.2358	.2262	.2116	.2215
.7	.1578	.1526	.1357	.1449
.8	.0906	.0942	.0773	.0851
.9	.0500	.0509	.0360	.0417

TABLE VI
EFFICIENCY OF REGRESSION COEFFICIENT ESTIMATES

Sample Size 50

ρ	MSEβ (DW) / MSEβ (OLS)	MSEβ (TN) / MSEβ (OLS)	MSEβ (PW) / MSEβ (OLS)	MSEβ (BM) / MSEβ (OLS)
.2	1.073	1.041	1.046	1.058
.3	.9985	.9482	.9714	.9562
.4	.8850	.8255	.8635	.8255
.5	.7452	.6859	.7282	.6825
.6	.5920	.5420	.5870	.5406
.7	.4366	.4020	.4453	.4020
.8	.2889	.2690	.3067	.2700
.9	.1589	.1505	.1738	.1505

APPENDIX
PROGRAM LISTINGS

This appendix contains listings of the programs utilized in the analysis performed herein. All of the functions are written in FORTRAN and contain the necessary documentation. The Monte Carlo simulation was performed using the Advanced Simulation and Statistics Package [Ref. 12] developed by P. A. Lewis. Since the package only allows for the simultaneous comparision of 3 estimators, 2 functions were developed for each sample size. The first, SIMS generates estimates for Durbin-Watson, Theil-Nagar, and Prais-Winsten for a sample size of 20. SIMSA meanwhile, generates estimates for Beach-MacKinnon for the identical sample size. Routines for Durbin-Watson and Theil-Nagar were included in SIMSA to ensure the results were comparable to SIMS. SIMSB and SIMSC perform in a similar fashion for sample size of 50 and therefore were not included. The Advanced Simulation and Statistics Package computes the mean square error of $\hat{\rho}$ for each estimator automatically. The mean square error for the β estimates was accomplished by the MSEB function.

```
                SIMS
      DIMENSION EHAT(20)
      COMMON /MYDATA/ K,T,ANS,Y1,X
      COMMON /DATA1/ IX1A,RHO
      REAL*4 Y(5000),YMIN,YMAX,PMEAN(3)
      CHARACTER*80 T1,T2,T3
      INTEGER N,M,NE(8),L,D,RG,SEI,SVS,NEST,NSR,IX1,IX2,IX3
      EXTERNAL  DATGEN, DURWAT, BEAMAC, PRAWIN, LSEB, DCALC, TRANSF
      EXTERNAL LNORM,SIMTBD,GMPRD
    . NR=20
      T=20
      K=2
C
C
```

```
C
       OPEN (UNIT=19,FILE='MONICA')
C     OPEN (UNIT=21,FILE='MARGE')
C     OPEN (UNIT=51,FILE='AMBROSE')
C     OPEN (UNIT=41,FILE='DAT2')
       OPEN (UNIT=61,FILE='DAT3')
       READ (19,*) ANS
10    READ(19,*,END=999) N,M,L,D,RG,SEI,SVS,NEST,NSR
       READ(19,*)YMIN,YMAX
       READ(19,*) (NE(I),I=1,L)
       READ(19,120) IX1,IX2,IX3
120   FORMAT(I5,1X,I5,1X,I5)
       READ (19,115) T1
115   FORMAT(A80)
       READ(19,115) T2
       READ (19,115)T3
       READ(19,*) (PMEAN(I),I=1,3)
       READ(19,*) RHO
       READ(19,61)IX1A
61    FORMAT(I5)
C
C     CALL FOR SIMTBD
C
       CALL SIMTBD (IX1,IX2,IX3,Y,N,M,NE,L,D,NSR,RG,SEI,SVS,
      *YMIN,YMAX,NEST,DATGEN,DURWAT,T1,DATGEN,BEAMAC,T2,DATGEN,PRAWIN,T3,
      *PMEAN)
       GO TO 10
999   WRITE(6,*)'END OF DATA INPUT'
       STOP
       END
C     ****************************************************************
C     *************  DATA GENERATION SUBROUTINE  **********************
C     *****************************************************************
       SUBROUTINE DATGEN (IX1,EHAT,NR)
       DIMENSION BHAT(2),YSTAR(20),R2(20),U(20),
```

```
     *E(20),YHAT(20),EHAT(20),XSTAR(20,2)
     *,Y1(20),X(20,2),V(20)
      COMMON /MYDATA/ K,T,ANS,Y1,X
      COMMON /DATA1/ IX1A,RHO
      INTEGER IX1,IX1A,NR
C
C
C     GENERATE THE RANDOM ERROR
C
      CALL SNOR (IX1,U,NR,1,0)
C
C     ADJUST THE VARIANCE OF R. E. IAW BEACH AND MACKINNON(1978)
      DO 38 I=1,T
         V(I)=U(I)*.06
38    CONTINUE
C
C     GENERATE THE ERROR FOR THE STAND LINEAR MODEL
C
      E(1)=V(1)/(1-(RHO**2))**0.5
      DO 31 J=2,T
         E(J)=RHO*E(J-1)+V(J)
31    CONTINUE
C
C
C     GENERATE THE EXPLANATORY VARIABLES IAW RAO AND GRILITCHES (1969)
C
      DO 32 I=1,20
         X(I,1)=1
32    CONTINUE
C     CHANGE IX1 IN ORDER TO AVIOD COLLINEARITY
C     IX1A=IX1+19
      CALL SNOR(IX1A,R2,NR,1,0)
      DO 33 J=1,20
         X(J,2)=R2(J)*.25
33    CONTINUE
```

```
C
C
C     THE TRUE BETA EQUALS 1,1
C
C
C     GENERATE THE INDEPENDENT VARIABLE
C
       DO 35 I=1,20
          Y1(I)=(X(I,1)+X(I,2))+E(I)
35    CONTINUE
C
C     GENERATE THE LEAST SQUARES ESTIMATOR FOR BETA
C
       CALL LSEB(X,Y1,BHAT)
C     PRINT LSEB TO A FILE
       IF(ANS .EQ. 2) WRITE(61,201)BHAT
201   FORMAT(F11.8,2X,F11.8)
C
C     GENERATE YHAT
C
       DO 100 I=1,20
       YHAT(I)=X(I,1)*BHAT(1)+X(I,2)*BHAT(2)
100   CONTINUE
C
C
C     GENERATE EHAT
C
       DO 50 I=1,20
          EHAT(I)=Y1(I)-YHAT(I)
50    CONTINUE
C
C
       RETURN
       END
C     ********************  DURBIN WATSON  **************************
```

```
C     THIS FUNCTION COMPUTES THE DURBIN-WATSON ESTIMATE OF RHO
       REAL FUNCTION DURWAT (EHAT,NR,WI)
       DIMENSION EHAT(20),X(20,2),Y1(20),XSTAR1(20,2),YSTAR1(20),BHAT1(2)
       COMMON /MYDATA/ K,T,ANS,Y1,X
       CALL DCALC (EHAT,T,D)
       DURWAT=1-D/2
C
       CALL TRANSF(X,Y1,DURWAT,XSTAR1,YSTAR1)
       CALL LSEB (XSTAR1,YSTAR1,BHAT1)
       IF (ANS .EQ. 1 ) WRITE(21,701) BHAT1
701   FORMAT(F11.8,2X,F11.8)
C
C
C
       RETURN
       END
C
C
C     ********************  THEIL NAGAR  ******************************
C     THIS FUNCTION COMPUTES THE THEIL-NAGAR ESTIMATE OF RHO
C     REAL FUNCTION THENAG (EHAT,NR,WI)
       DIMENSION EHAT(20),YSTAR2(20),XSTAR2(20,2),BHAT2(2)
      *,Y1(20),X(20,2)
       COMMON /MYDATA/ K,T,ANS,Y1,X
       CALL DCALC (EHAT,T,D)
       THENAG=((T**2)*(1-D/2)+K**2)/(T**2-K**2)
       CALL TRANSF(X,Y1,THENAG,XSTAR2,YSTAR2)
       CALL LSEB (XSTAR2,YSTAR2,BHAT2)
       IF (ANS .EQ. 1 ) WRITE(31,801) BHAT2
801   FORMAT(F11.8,2X,F11.8)
       RETURN
       END
C     ***********************  PRAIS WINSTEN  *************************
C     THIS FUNCTION COMPUTES THE PRAIS-WINSTEN ESTIMATE OF RHO
       REAL FUNCTION PRAWIN(EHAT,NR,WI)
```

```
      DIMENSION EHAT3(20),YHAT3(20),YSTAR3(20),BHAT3(2),
     *EHAT(20),XSTAR3(20,2)
     *,Y1(20),X(20,2)
      COMMON /MYDATA/ K,T,ANS,Y1,X
      N=0
      RHO3=0
98    N=N+1
      CALL TRANSF (X,Y1,RHO3,XSTAR3,YSTAR3)
      CALL LSEB (XSTAR3,YSTAR3,BHAT3)
C     GENERATE YHAT3
      DO 83 I=1,20
         YHAT3(I)=X(I,1)*BHAT3(1)+X(I,2)*BHAT3(2)
83    CONTINUE
      DO 4 I=1,T
         EHAT3(I)=Y1(I)-YHAT3(I)
4     CONTINUE
C
      RHONUM=0
      RHODEN=0
      DO 5 I=2,T
         RHONUM=RHONUM+(EHAT3(I)*EHAT3(I-1))
5     CONTINUE
C
      DO 6 I=2,T-1
         RHODEN=RHODEN+(EHAT3(I)**2)
6     CONTINUE
      PRAWIN=RHONUM/RHODEN
C     CHECK FOR PRAWIN WHICH ARE OUT OF BOUNDS
      IF(PRAWIN.GE.1)THEN
         PRAWIN=0.99999
      ELSE IF (PRAWIN.LE.-1)THEN
         PRAWIN=-0.99999
      END IF
C     COMPARISION OF RHO3 AND PRAWIN IF DIFF .LT. 0.0001 THEN END
      IF(ABS(RHO3-PRAWIN).GT..0001)THEN
```

```
          RHO3=PRAWIN
          GO TO 98
       ELSE
          PRAWIN=PRAWIN
       END IF
C     IF (ANS .EQ. 1 ) WRITE(41,901) BHAT3
C01   FORMAT(F11.8,2X,F11.8)
       RETURN
       END
C
C     THE FOLLOWING SUBROUTINES AID IN THE COMPUTATION OF THE FOUR
C     ESTIMATORS OF RHO.
C     *************  SUBROUTINE LSEB  ******************************
C     SUBROUTINE LSEB WILL COMPUTE THE LSE OF B
C
       SUBROUTINE LSEB(X,Y1,BHAT)
       DIMENSION BHAT(2),Y1(20),X(20,2),XTRNSP(2,20),XI(2,2),H(2,20),
      *XPRIX(2,2)
C     X TRANSPOSE
       DO 40 I=1,20
          DO 41 J=1,2
                XTRNSP(J,I)=X(I,J)
41        CONTINUE
40    CONTINUE
C     MULTIPLY X TRANSPOSE AND X
       CALL GMPRD(XTRNSP,X,XPRIX,2,20,2)
C     CALCULATE INVERSE OF X PRIME X
       DETR=1/(XPRIX(1,1)*XPRIX(2,2)-XPRIX(1,2)*XPRIX(2,1))
       XI(1,1)=DETR*XPRIX(2,2)
       XI(1,2)=DETR*(-XPRIX(1,2))
       XI(2,1)=DETR*(-XPRIX(2,1))
       XI(2,2)=DETR*XPRIX(1,1)
C     MULTIPLY INVERSE AND TRANSPOSE
       CALL GMPRD(XI,XTRNSP,H,2,2,20)
       DO 99 I=1,2
```

```
      BHAT(I)=H(I,1)*Y1(1)+H(I,2)*Y1(2)+H(I,3)*Y1(3)
     *+H(I,4)*Y1(4)+H(I,5)*Y1(5)
     *+H(I,6)*Y1(6)+H(I,7)*Y1(7)+H(I,8)*Y1(8)+H(I,9)*Y1(9)
     *+H(I,10)*Y1(10)+
     *H(I,11)*Y1(11)+H(I,12)*Y1(12)+H(I,13)*Y1(13)+H(I,14)*Y1(14)
     *+H(I,15)*Y1(15)+
     *H(I,16)*Y1(16)+H(I,17)*Y1(17)+H(I,18)*Y1(18)+H(I,19)*Y1(19)+
     *H(I,20)*Y1(20)
99   CONTINUE
      RETURN
      END
C    *************  SUBROUTINE DCALC  *****************************
C    SUBROUTINE DCALC WILL COMPUTE THE DURBIN STATISTIC D
C
      SUBROUTINE DCALC(EHAT,T,D)
      DIMENSION D1(20),D2(20),EHAT(20)
      DNUM=0
      DDEN=0
      DO 1 I=2,T
         D1(I-1)=(EHAT(I)-EHAT(I-1))**2
         DNUM=DNUM+D1(I-1)
1    CONTINUE
      DO 2 J=1,T
         D2(J)=EHAT(J)**2
         DDEN=DDEN+D2(J)
2    CONTINUE
      D=DNUM/DDEN
      RETURN
      END
C
C    *****************  SUBROUTINE TRANSF  *********************
C
C    SUBROUTINE TRANSF IS DESIGNED TO TRANSFORM THE X'S AND Y'S
C    ACCORDING TO THE LEAST SQUARES RULE.
      SUBROUTINE TRANSF(X,Y1,RHOHAT,XSTAR,YSTAR)
```

```
      DIMENSION Y1(20),YSTAR(20),X(20,2),XSTAR(20,2)
      K=2
      T=20
C     Y TRANSFORM
      YSTAR(1)=((1-(RHOHAT**2))**0.5)*Y1(1)
      DO 7 I=2,20
         YSTAR(I)=Y1(I)-(RHOHAT*Y1(I-1))
7     CONTINUE
C     X TRANSFORM
      XSTAR(1,1)=(1-(RHOHAT**2))**0.5
      DO 9 J=2,K
         XSTAR(1,J)=((1-(RHOHAT**2))**0.5)*X(1,J)
9     CONTINUE
      DO 11 L=2,T
         XSTAR(L,1)=1-RHOHAT
11    CONTINUE
      DO 12 I=2,T
         DO 13 J=2,K
               XSTAR(I,J)=X(I,J)-RHOHAT*X(I-1,J)
13       CONTINUE
12    CONTINUE
      RETURN
      END
```

```
               SIMSA
C     THE PURPOSE OF THIS PROGRAM IS TO RUN COMPUTE THE  FOLLOWING
C     ESTIMATORS (DW TN BM) FOR A SAMPLE SIZE OF 20
       DIMENSION EHAT(20)
       COMMON /MYDATA/ K,T,ANS,Y1,X
       COMMON /DATA1/ IX1A,RHO
       REAL*4 Y(5000),YMIN,YMAX,PMEAN(3)
       CHARACTER*80 T1,T2,T3
       INTEGER N,M,NE(8),L,D,RG,SEI,SVS,NEST,NSR,IX1,IX2,IX3
       EXTERNAL  DATGEN, DURWAT, THENAG, BEAMAC, LSEB, DCALC, TRANSF
       EXTERNAL LNORM,SIMTBD,GMPRD
       NR=20
       T=20
       K=2
C
C
C
       OPEN (UNIT=19,FILE='MONICA')
       OPEN (UNIT=51,FILE='AMBROSE')
       READ (19,*) ANS
10    READ(19,*,END=999) N,M,L,D,RG,SEI,SVS,NEST,NSR
       READ(19,*)YMIN,YMAX
       READ(19,*) (NE(I),I=1,L)
       WRITE (22,105) (NE(I),I=1,L)
105   FORMAT(8I4)
       READ(19,120) IX1,IX2,IX3
120   FORMAT(I5,1X,I5,1X,I5)
       READ (19,115) T1
115   FORMAT(A80)
       READ(19,115) T2
       READ (19,115)T3
       READ(19,*) (PMEAN(I),I=1,3)
       READ(19,*) RHO
       READ(19,61)IX1A
61    FORMAT(I5)
```

```
C
C     CALL FOR SIMTBD
C
       CALL SIMTBD (IX1,IX2,IX3,Y,N,M,NE,L,D,NSR,RG,SEI,SVS,
     *YMIN,YMAX,NEST,DATGEN,DURWAT,T1,DATGEN,THENAG,T2,DATGEN,BEAMAC,T3,
     *PMEAN)
       GO TO 10
999   WRITE(6,*)'END OF DATA INPUT'
       STOP
       END
C     ****************************************************************
C     *************  DATA GENERATION SUBROUTINE  **********************
C     *****************************************************************
       SUBROUTINE DATGEN (IX1,EHAT,NR)
       DIMENSION BHAT(2),YSTAR(20),R2(20),U(20),
     *E(20),YHAT(20),EHAT(20),XSTAR(20,2)
     *,Y1(20),X(20,2)
       COMMON /MYDATA/ K,T,ANS,Y1,X
       COMMON /DATA1/ IX1A,RHO
       INTEGER IX1,IX1A,NR
C
C
C     GENERATE THE RANDOM ERROR
C
       CALL SNOR (IX1,U,NR,1,0)
C
C
C     GENERATE THE ERROR FOR THE STAND LINEAR MODEL
C
       E(1)=U(1)/(1-(RHO**2))**0.5
       DO 31 J=2,20
          E(J)=RHO*E(J-1)+U(J)
31    CONTINUE
C
C
```

```
C     GENERATE THE EXPLANATORY VARIABLES IAW RAO AND GRILITCHES (1969)
C
       DO 32 I=1,20
          X(I,1)=1
32    CONTINUE
C     CHANGE IX1 IN ORDER TO AVIOD COLLINEARITY
C     IX1A=IX1+19
       CALL SNOR(IX1A,R2,NR,1,0)
       DO 33 J=1,20
          X(J,2)=R2(J)*.25
33    CONTINUE
C
C
C     THE TRUE BETA EQUALS 1,1
C
C
C     GENERATE THE INDEPENDENT VARIABLE
C
       DO 35 I=1,20
          Y1(I)=(X(I,1)+X(I,2))+E(I)
35    CONTINUE
C
C
C     GENERATE YHAT
C
       CALL LSEB(X,Y1,BHAT)
C     BHAT(1)=1.3
C     BHAT(2)=1.1
       DO 100 I=1,20
       YHAT(I)=X(I,1)*BHAT(1)+X(I,2)*BHAT(2)
100   CONTINUE
C
C
C     GENERATE EHAT
C
```

```
      DO 50 I=1,20
         EHAT(I)=Y1(I)-YHAT(I)
50    CONTINUE
C
C
      RETURN
      END
C     *********************  DURBIN WATSON  ***************************
C
      REAL FUNCTION DURWAT (EHAT,NR,WI)
      DIMENSION EHAT(20),X(20,2),Y1(20),XSTAR1(20,2),YSTAR1(20),BHAT1(2)
      COMMON /MYDATA/ K,T,ANS,Y1,X
      CALL DCALC (EHAT,T,D)
      DURWAT=1-D/2
      CALL TRANSF(X,Y1,DURWAT,XSTAR1,YSTAR1)
      CALL LSEB (XSTAR1,YSTAR1,BHAT1)
C
C
C
      RETURN
      END
C
C
C     ********************  THEIL NAGAR  ******************************
C
      REAL FUNCTION THENAG (EHAT,NR,WI)
      DIMENSION EHAT(20),YSTAR2(20),XSTAR2(20,2),BHAT2(2)
     *,Y1(20),X(20,2)
      COMMON /MYDATA/ K,T,ANS,Y1,X
      CALL DCALC (EHAT,T,D)
      THENAG=((T**2)*(1-D/2)+K**2)/(T**2-K**2)
      RETURN
      END
C
C     ***********************  BEACH MACKINNON *************************
```

```
C
       REAL FUNCTION BEAMAC(EHAT,NR,WI)
       DIMENSION EHAT4(20),YHAT4(20),YSTAR4(20),BHAT4(2),
      *Y1(20),EHAT(20),X(20,2),XSTAR4(20,2)
       COMMON /MYDATA/ K,T,ANS,Y1,X
       N=0
       RHO4=0
98    N=N+1
       CALL TRANSF (X,Y1,RHO4,XSTAR4,YSTAR4)
       CALL LSEB (XSTAR4,YSTAR4,BHAT4)
C     BHAT4(1)=1.0
C     BHAT4(2)=1.0
C     GENERATE YHAT4
       DO 83 I=1,20
          YHAT4(I)=X(I,1)*BHAT4(1)+X(I,2)*BHAT4(2)
83    CONTINUE
       DO 4 I=1,T
          EHAT4(I)=Y1(I)-YHAT4(I)
4     CONTINUE
       SUM3=0
       SUM2=0
       SUM1=0
       DO 71 I=2,T
          SUM1=SUM1+(EHAT4(I)*EHAT4(I-1))
71    CONTINUE
C
       DO 72 I=2,T
          SUM2=SUM2+(EHAT4(I-1)**2)
72    CONTINUE
C
       DO 73 I=2,T
          SUM3=SUM3+(EHAT4(I)**2)
73    CONTINUE
C
       DENOM=(T-1)*(SUM2-(EHAT4(1)**2))
```

```
C
      A=(-(T-2)*SUM1)/DENOM
C
      B=(((T-1)*(EHAT4(1)**2))-(T*SUM2)-SUM3)/DENOM
C
      C=(T*SUM1)/DENOM
C
      SMALP=B-((A**2)/3)
C
      SMALQ=C-((A*B)/3)+((2*(A**3))/27)
C
      THETA=ACOS((SMALQ*(27**.5))/(2*SMALP*((-SMALP)**0.5)))
C
C     BEAMAC IS THE ITERATIVE RHO FOR THIS PROCEEDURE
      BEAMAC=(-2*((-SMALP/3)**0.5))*COS((THETA/3)+(3.1412/3))-(A/3)
C     CHECK FOR BEAMAC WHICH ARE OUT OF BOUNDS
      IF(BEAMAC.GE.1)THEN
         BEAMAC=0.99999
      ELSE IF (BEAMAC.LE.-1)THEN
         BEAMAC=-0.99999
      END IF
C     COMPARISION OF RHO4 AND BEAMAC IF DIFF .LT. 0.0001 THEN END
      IF(ABS(RHO4-BEAMAC).GT..0001)THEN
         RHO4=BEAMAC
         GO TO 98
      ELSE
         BEAMAC=BEAMAC
      END IF
      IF (ANS .EQ. 2) WRITE (51,901) BEAMAC
901   FORMAT(F15.11)
      RETURN
      END
C
C     THE FOLLOWING SUBROUTINES AID IN THE COMPUTATION OF THE FOUR
C     ESTIMATORS OF RHO.
```

```
C      *************  SUBROUTINE LSEB  *******************************
C      SUBROUTINE LSEB WILL COMPUTE THE LSE OF B
C
       SUBROUTINE LSEB(X,Y1,BHAT)
       DIMENSION BHAT(2),Y1(20),X(20,2),XTRNSP(2,20),XI(2,2),H(2,20),
      *XPRIX(2,2)
C      X TRANSPOSE
       DO 40 I=1,20
          DO 41 J=1,2
               XTRNSP(J,I)=X(I,J)
41       CONTINUE
40     CONTINUE
C      MULTIPLY X TRANSPOSE AND X
       CALL GMPRD(XTRNSP,X,XPRIX,2,20,2)
C      CALCULATE INVERSE OF X PRIME X
       DETR=1/(XPRIX(1,1)*XPRIX(2,2)-XPRIX(1,2)*XPRIX(2,1))
       XI(1,1)=DETR*XPRIX(2,2)
       XI(1,2)=DETR*(-XPRIX(1,2))
       XI(2,1)=DETR*(-XPRIX(2,1))
       XI(2,2)=DETR*XPRIX(1,1)
C      MULTIPLY INVERSE AND TRANSPOSE
       CALL GMPRD(XI,XTRNSP,H,2,2,20)
       DO 99 I=1,2
       BHAT(I)=H(I,1)*Y1(1)+H(I,2)*Y1(2)+H(I,3)*Y1(3)
      *+H(I,4)*Y1(4)+H(I,5)*Y1(5)
      *+H(I,6)*Y1(6)+H(I,7)*Y1(7)+H(I,8)*Y1(8)+H(I,9)*Y1(9)
      *+H(I,10)*Y1(10)+
      *H(I,11)*Y1(11)+H(I,12)*Y1(12)+H(I,13)*Y1(13)+H(I,14)*Y1(14)
      *+H(I,15)*Y1(15)+
      *H(I,16)*Y1(16)+H(I,17)*Y1(17)+H(I,18)*Y1(18)+H(I,19)*Y1(19)+
      *H(I,20)*Y1(20)
99     CONTINUE
       RETURN
       END
C      *************  SUBROUTINE DCALC  *****************************
```

```
C     SUBROUTINE DCALC WILL COMPUTE THE DURBIN STATISTIC D
C
       SUBROUTINE DCALC(EHAT,T,D)
       DIMENSION D1(20),D2(20),EHAT(20)
       DNUM=0
       DDEN=0
       DO 1 I=2,T
          D1(I-1)=(EHAT(I)-EHAT(I-1))**2
          DNUM=DNUM+D1(I-1)
1     CONTINUE
       DO 2 J=1,T
          D2(J)=EHAT(J)**2
          DDEN=DDEN+D2(J)
2     CONTINUE
       D=DNUM/DDEN
       RETURN
       END
C
C     *****************  SUBROUTINE TRANSF  **********************
C
C     SUBROUTINE TRANSF IS DESIGNED TO TRANSFORM THE X'S AND Y'S
C     ACCORDING TO THE LEAST SQUARES RULE.
       SUBROUTINE TRANSF(X,Y1,RHOHAT,XSTAR,YSTAR)
       DIMENSION Y1(20),YSTAR(20),X(20,2),XSTAR(20,2)
       K=2
       T=20
C     Y TRANSFORM
       YSTAR(1)=((1-(RHOHAT**2))**0.5)*Y1(1)
       DO 7 I=2,20
          YSTAR(I)=Y1(I)-(RHOHAT*Y1(I-1))
7     CONTINUE
C     X TRANSFORM
       XSTAR(1,1)=(1-(RHOHAT**2))**0.5
       DO 9 J=2,K
          XSTAR(1,J)=((1-(RHOHAT**2))**0.5)*X(1,J)
```

```
9     CONTINUE
       DO 11 L=2,T
          XSTAR(L,1)=1-RHOHAT
11    CONTINUE
       DO 12 I=2,T
          DO 13 J=2,K
               XSTAR(I,J)=X(I,J)-RHOHAT*X(I-1,J)
13       CONTINUE
12    CONTINUE
       RETURN
       END
```

```
                MSEB
C     THIS PROGRAM IS DESIGNED TO CALCULATE THE MEAN SQUARE ERROR OF
C     THE BETA VECTOR
       DIMENSION B1(5000),B2(5000),B3(5000),B4(5000),B5(5000),B6(5000),
      *B7(5000),B8(5000),B9(5000),B10(5000),BX(5000),BY(5000)
       OPEN (UNIT=21,FILE='DAT1')
       OPEN (UNIT=31,FILE='DAT2')
       OPEN (UNIT=41,FILE='DAT3')
       OPEN (UNIT=51,FILE='DAT4')
       OPEN (UNIT=61,FILE='DAT5')
C
       COUNT=1000
       READ(21,900)(B1(I),B2(I), I=1,1000)
       CALL MSEBET (B1,B2,COUNT,XMSEDW)
       READ(31,900)(B3(I),B4(I), I=1,1000)
       CALL MSEBET (B3,B4,COUNT,XMSETN)
       READ(41,900)(B5(I),B6(I), I=1,1000)
       CALL MSEBET (B5,B6,COUNT,XMSEPW)
       READ(51,900)(B7(I),B8(I), I=1,1000)
       CALL MSEBET (B7,B8,COUNT,XMSEBM)
       READ(61,900)(B9(I),B10(I), I=1,1000)
       CALL MSEBET (B9,B10,COUNT,XMSEOLS)
900   FORMAT (F11.8,2X,F11.8)
       WRITE(6,*)'MSEDW'
       WRITE(6,*)XMSEDW
C
       WRITE(6,*)'MSETN'
       WRITE(6,*)XMSETN
C
       WRITE(6,*)'MSEPW'
       WRITE(6,*)XMSEPW
C
       WRITE(6,*)'MSEBM'
       WRITE(6,*)XMSEBM
C
```

```
      WRITE(6,*)'MSELS'
      WRITE(6,*)XMSELS
      STOP
      END
C     **************** SUBROUTINE MSEBET  ****************************
C
      SUBROUTINE MSEBET(BX,BY,AN,XMSEB)
      DIMENSION BX(5000),BY(5000),SUM(5000)
      PLACE=0
      DO 901 I=1,AN
      SUM(I)=((BX(I)-1)*(BY(I)-1))**2
      PLACE=PLACE + SUM(I)
901   CONTINUE
      XMSEB=PLACE/AN
      RETURN
      END
```

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